A Real Time Hybrid Simulation (RTHS) DEMO for a Single Story Steel Structure with Base Isolation[a]

Daniel Gomez[1], Rachel Chicchi[2], Scott Wonderly[3], Shirley J Dyke[4]
[1,2,3,4] Purdue University, West Lafayette, IN 47907
Email: dgomezp@purdue.edu

Contents

Abstract

This document describes the set-up of a simple RTHS simulation with base isolation and is accompanied by two files BI_RTHS_simul.mdl and BI_RTHS_DM.m. BI_RTHS_simul.mdl is a Matlab Simulink® code that uses a block environment to simulate the model response. This simulation builds upon "A Real Time Hybrid Simulation (RTHS) DEMO for a Single Story Steel Structure" (Ou, https://nees.org/resources/6984), which will be referred to in this document as the "original demo." In this new model, a base isolator is applied to a single story steel structure with the same mass, stiffness, damping ratio as the original demo. The structure is partitioned into one numerical substructure (the steel structure) and one experimental substructure (the base isolator).

The purpose of preparing this demo example is to help beginners understand the RTHS concept and build upon the single degree of freedom RTHS system in the original demo for multi degree of freedom systems. It also allows users the option to vary base isolator parameters to determine its effects on the response of the structure.

Keyword: Hybrid Simulation, Real Time Hybrid Simulation, Demo code, Base Isolation

[a] **NOTE: This code is the simulation of RTHS and is not intended to run on a real time operating system.**

1 Concept of RTHS

The use of traditional techniques such as the shake table or quasi-static tests are often used to develop and validate different configurations and design strategies, and improve the seismic engineering design process. Unfortunately, physically testing the performance of entire buildings systems in laboratories can be impractical due to scale and costs involved. However, laboratories can generally accommodate testing of critical components of structures, and calculate the responses of a structure's noncritical or better-modeled components through numerical simulation (Seismic Waves, 2013). This is the concept behind Hybrid Simulation that has been broadly used in very different applications in civil engineering, where the critical components of the structural system under evaluation are physically tested. Oftentimes, the test is rate dependent and the simulation must be executed in real-time. It is in this case that Real Time Hybrid Simulation (RTHS) methods provide an alternate approach to evaluating structural / rate-dependent systems under actual dynamic and inertial conditions, without the need for full-scale structural testing (Castaneda et al., 2012). Figure 1 illustrates the seismic study conducted for this demo, which consists of a single story steel structure with a base isolator.

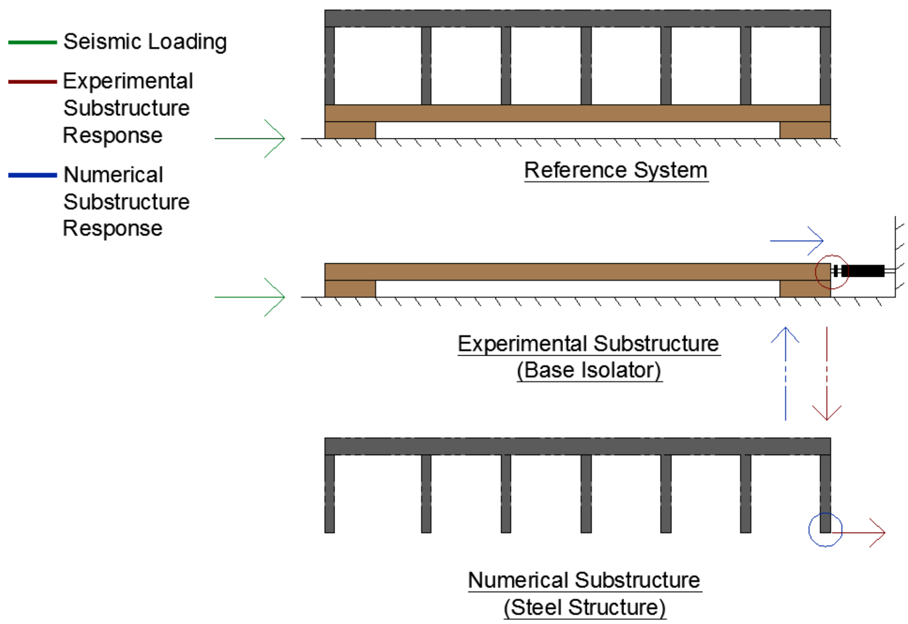

Figure 1. Concept of RTHS

The RTHS cycle is much like that described in the original demo, by Ou (2013). However, the order has been altered to achieve the goals of this particular test. The original demo (without the base isolator) began with an earthquake acceleration input to the numerical structure, which commanded a displacement input to the actuator to generate a displacement in the experimental structure. Then, the experimental structure's reaction sent a feedback force back into the numerical structure to complete the cycle. Instead, this demo sends earthquake ground displacements and velocities into the base isolator (experimental substructure). The experimental responses (displacement and velocity) with the actuator displacement are transferred to the numerical substructure. As in the original demo, the experimental structure that is normally tested in a lab using shake tables or actuators is simulated using Matlab and Simulink software.

2 Structure of demo code

The Simulink file, BI_RTHS_simul.mdl is compatible with 6.6/R2007a version or later. The layout for this block-based simulation environment is shown below. The BI_RTHS_DM.m file defines the structural parameters and executes the Simulink code.

BI_RTHS_simul.mdl is divided into three sections that reflect the basic layout of the original demo. The first portion is the reference system (in yellow), which is the earthquake response of the total structure without partitioning. This will serve as a baseline comparison to the RTHS method. The second section, comprised of an experimental substructure (dark red) and a numerical substructure (grey) is an idealized RTHS case that does not consider any actuator dynamics or controls in the experimental system. Finally, the last section represents the RTHS case that considers the dynamics of an actuator (green) between the experimental and numerical structure (see Figure No. 2).

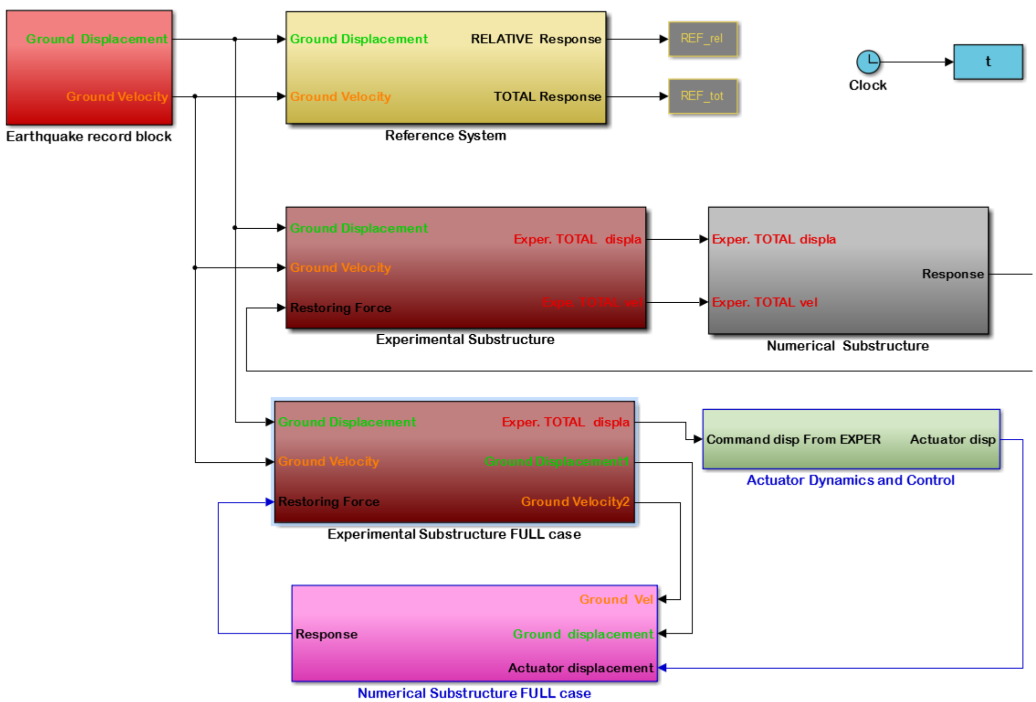

Figure 2. BI_RTHS_simul.mdl Simulink Diagram

In the following sections, the components of the demo are explained in more detail. Refer to the read me.txt file for a complete list of file names and descriptions.

2.1 Structural properties and state space form

The one story steel structure in the original demo is used again in this demo and its properties are:

$$M_N = 155.6 \frac{N}{m/s^2}, K_N = 1.005 \times 10^5 \frac{N}{m}, \zeta_N = 0.02$$

Where M_N, K_N, ζ_N are mass, stiffness, damping ratio of the one story steel (numerical) structure.

Additionally, this demo has the option of using three different options for the damping, mass, and stiffness in the base isolator level. These options can be seen on page 8. Because the base isolation system has very high damping properties and the steel structure has significantly less damping capabilities, the combined system would behave in a non-classical damping fashion. However, because the objective of this demo is limited to understanding base isolation and RTHS modeling, assuming structural linearity provides approximate results that would suffice for the above mentioned objectives.

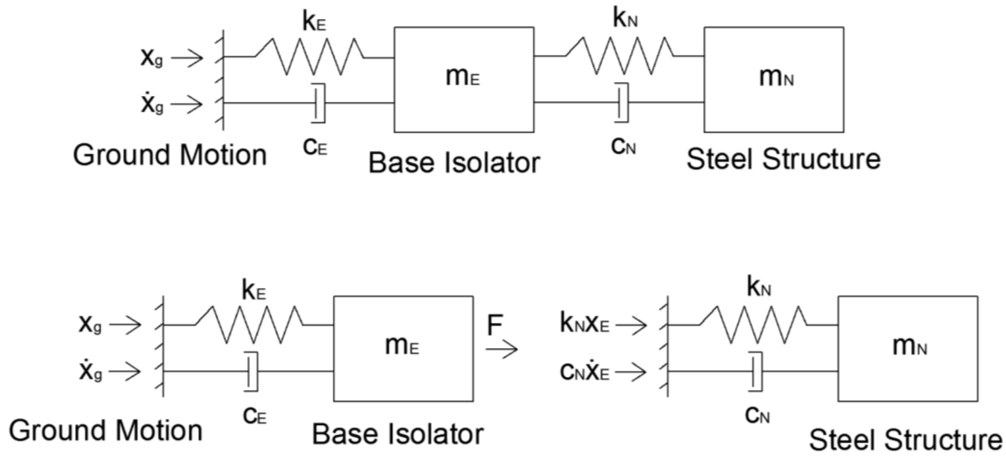

Figure 3. Model of the reference and partitioned system

Figure 3 shows the base isolator and the steel structure modeled as spring-mass-damper systems. It is also shown partitioned, which is necessary for analysis.

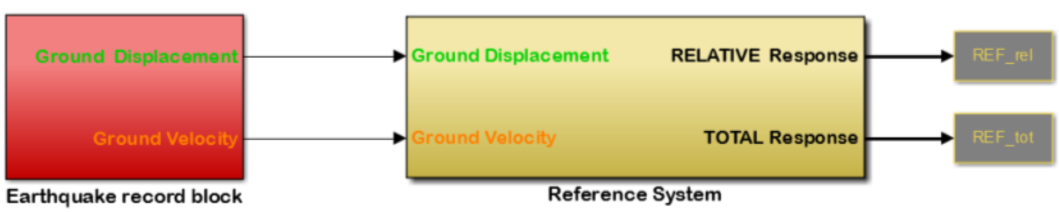

Figure 4. Entire structure response simulation (no partitioning)

Simulink performs the numerical integration and simulates a linear structural response under seismic loading through use of the state space form of the equation of motion. x_N and x_E represent the response of the experimental (base isolator) and numerical (steel structure) substructures, respectively. The equations of motions are:

$$m_E \ddot{x}_E + c_E \dot{x}_E + k_E x_E = F + k_E x_g + c_E \dot{x}_g \quad (1)$$

$$m_N \ddot{x}_N + c_N \dot{x}_N + k_N x_N = k_N x_E + c_N \dot{x}_E \quad (2)$$

$$F = k_N (x_N - x_E) + c_N (\dot{x}_N - \dot{x}_E) \quad (3)$$

$$\dot{x}_E = y_E \quad (4)$$

$$\dot{x}_N = y_N \quad (5)$$

Substituting in y and F values:

$$m_E \dot{y}_E + c_E y_E + k_E x_E = k_N (x_N - x_E) + c_N (y_N - y_E) + k_E x_g + c_E \dot{x}_g \tag{6}$$

$$m_N \dot{y}_N + c_N y_N + k_N x_N = k_N x_E + c_N y_E \tag{7}$$

These equations can be written as two first order state space equations:

$$\dot{z}_{Ref} = \begin{bmatrix} x_N \\ x_E \\ y_N \\ y_E \end{bmatrix} = A_{Ref} \begin{bmatrix} x_N \\ x_E \\ y_N \\ y_E \end{bmatrix} + B_{Ref} \begin{bmatrix} x_g \\ \dot{x}_g \end{bmatrix} \tag{8}$$

$$\ddot{z}_{Ref} = \begin{bmatrix} x_N \\ x_E \\ y_N \\ y_E \\ \dot{y}_N \\ \dot{y}_E \end{bmatrix} = C_{Ref} \begin{bmatrix} x_E \\ x_N \\ y_N \\ y_E \end{bmatrix} + D_{Ref} \begin{bmatrix} x_g \\ \dot{x}_g \end{bmatrix} \tag{9}$$

Where,

$$A_{Ref} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-k_N}{m_N} & \frac{k_N}{m_N} & \frac{-c_N}{m_N} & \frac{c_N}{m_N} \\ \frac{k_N}{m_E} & \frac{-(k_N+k_E)}{m_E} & \frac{c_N}{m_E} & \frac{-(c_N+c_E)}{m_E} \end{bmatrix} \; ; \; B_{Ref} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ (k_E/m_E) & (c_E/m_E) \end{bmatrix}$$

$$C_{Ref} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-k_N}{m_N} & \frac{k_N}{m_N} & \frac{-c_N}{m_N} & \frac{c_N}{m_N} \\ \frac{k_N}{m_E} & \frac{-(k_N+k_E)}{m_E} & \frac{c_N}{m_E} & \frac{-(c_N+c_E)}{m_E} \end{bmatrix} \; ; \; D_{Ref} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ (k_E/m_E) & (c_E/m_E) \end{bmatrix}$$

Eqns. (8) and (9) are the state space representation of Eqns (6) and (7), where x_g and $\dot{x}_g$, are the absolute earthquake displacement and velocity respectively, $\dot{z}_{Ref}$ is the state vector, and $\ddot{z}_{Ref}$ is the structural response vector for the entire structure.

The corresponding parameters are provided in the BI_RTHS_DM.m file.

```
% RTHS Reference system
% Input:   1) Ground Displacement, [m]
%          2) Ground Velocity,     [m/sec]
% Outputs: 1) Total displacement of ROOF
%          2) Total displacement of BI
%          3) Total velocity of ROOF
%          4) Total velocity of BI
%          5) Total acceleration of ROOF
%          6) Total acceleration of BI


% Steel Building (Numerical Substructure)
ndof = 2;
MN = M;
KN = K;
CN = C;

% Base Isolator (Experimental Substructure)
ME = m_b;
KE = k_b;
CE = c_b;
zeta_e = zeta_b;

% REFERENCE SYSTEM (Experimental and Numerical Substructure)
AA_REF=[zeros(ndof) eye(ndof); -KN/MN KN/MN -CN/MN CN/MN; KN/ME -(KN+KE)/ME CN/ME -↙
(CN+CE)/ME];
BB_REF=[zeros(ndof+1,2); KE/ME CE/ME];
CC_REF=[eye(ndof*2); -KN/MN KN/MN -CN/MN CN/MN; KN/ME -(KN+KE)/ME CN/ME -(CN+CE)/ME];
DD_REF=[zeros(ndof*2+1,2); KE/ME CE/ME];

sys_REF = ss(AA_REF,BB_REF,CC_REF,DD_REF);
```

2.2 RTHS partitioning

Because the structure and base isolator are combined to conform a coupled system, it was necessary to re-write the equations in order to separate the experimental and numerical systems, as illustrated in Figure 3. Simulink and the state space form of the equations of motion are used again for RTHS partitioning. In Figure 5, the building with its base isolation is shown in a simple scheme.

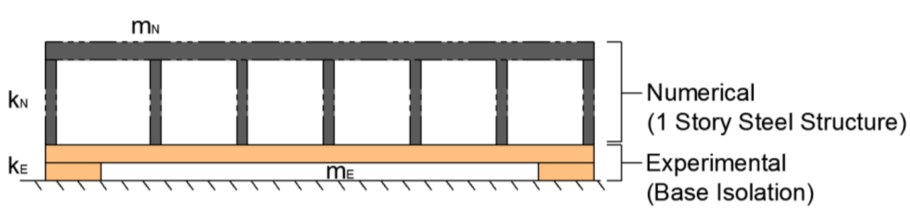

Figure 5. Base isolation case (Reference system)

Figure 6 shows the components in the RTHS ideal case. The ground displacement and velocity are the inputs in the experimental substructure (base isolation) and the one story steel structure is the numerical substructure.

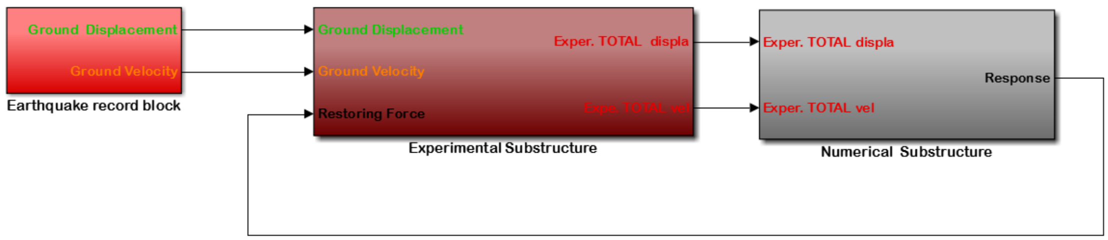

Figure 6. Ideal case RTHS

2.2.1 Experimental substructure

The experimental structure in this demonstration is the base isolator for the steel structure. After separating the base isolator from the system, the state space equations can formulated in the manner shown below.

Equation (1) represents the equation of motion for the experimental substructure. This structure can be written as two first order state space equations (see Equations 10 and 11).

$$\dot{\mathrm{z}}_\mathrm{E} = \begin{bmatrix} y_E \\ \dot{y}_E \end{bmatrix} = \mathrm{A}_\mathrm{E} \begin{bmatrix} x_E \\ y_E \end{bmatrix} + \mathrm{B}_\mathrm{E} \begin{bmatrix} x_g \\ \dot{x}_g \\ F \end{bmatrix} \tag{10}$$

$$\ddot{\mathrm{z}}_\mathrm{E} = \begin{bmatrix} x_E \\ y_E \\ \dot{y}_E \end{bmatrix} = \mathrm{C}_\mathrm{E} \begin{bmatrix} x_E \\ y_E \end{bmatrix} + \mathrm{D}_\mathrm{E} \begin{bmatrix} x_g \\ \dot{x}_g \\ F \end{bmatrix} \tag{11}$$

where,

$$\mathrm{A}_\mathrm{E} = \begin{bmatrix} 0 & 1 \\ -(k_E/m_E) & -(c_E/m_E) \end{bmatrix} \;;\; \mathrm{B}_\mathrm{E} = \begin{bmatrix} 0 & 0 & 0 \\ (k_E/m_E) & (c_E/m_E) & (1/m_E) \end{bmatrix}$$

$$\mathrm{C}_\mathrm{E} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -(k_E/m_E) & -(c_E/m_E) \end{bmatrix} \;;\; \mathrm{D}_\mathrm{E} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ (k_E/m_E) & (c_E/m_E) & (1/m_E) \end{bmatrix}$$

As before, the subscript, *E*, represents the experimental portion, *F* represents the restoring force, and x_g and $\dot{x}_g$ represent the earthquake displacement and velocity, respectively.

```
% STATE SPACE MODEL(Experimental Substructure)
AA_E = [0 1; -KE/ME -CE/ME];
BB_E = [0 0 0; KE/ME CE/ME 1/ME];
CC_E = [1 0; 0 1;-KE/ME -CE/ME];
DD_E = [zeros(2,3); KE/ME CE/ME 1/ME];
sys_E = ss(AA_E,BB_E,CC_E,DD_E);
```

Base isolation is most effective if the fundamental period of the isolation system is much larger than the steel structure above. Increasing the period of the structure reduces the pseudo-accelerations and, therefore, decreases the base shear and earthquake forces exhibited in the structure. High deformations in the isolation system result in smaller deformations in the steel structure, as the isolation system absorbs most of the energy transmitted from the ground motion. The experimental system here has three options for system parameters. This allows the user to select various properties to utilize, in order to examine different behaviors in the system. The three options for this base isolation system are:

Option	Mass [N/(m/s^2)]	Stiffness [N/m]	Zeta
1	10	K/200	0.5
2	10	K/20	0.5
3	10,0000	K/2	0.5

Table 1. Base Isolation parameter options

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%        Base Isolation Information        %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Option = 3;

switch Option
case 1
     m_b =  10;
     k_b =  K/200;
     zeta_b = .5;
     c_b    = 2*zeta_b*sqrt(k_b*m_b);
case 2
     m_b =  10;
     k_b =  K/20;
     zeta_b = .5;
     c_b = 2*zeta_b*sqrt(k_b*m_b);
case 3
     m_b =  10000;
     k_b =  K/2;
     zeta_b = .5;
     c_b= 2*zeta_b*sqrt(k_b*m_b);
end
```

The three options were chosen to show how changing the properties, most notably the stiffness, affect the response of a structure with base isolation.

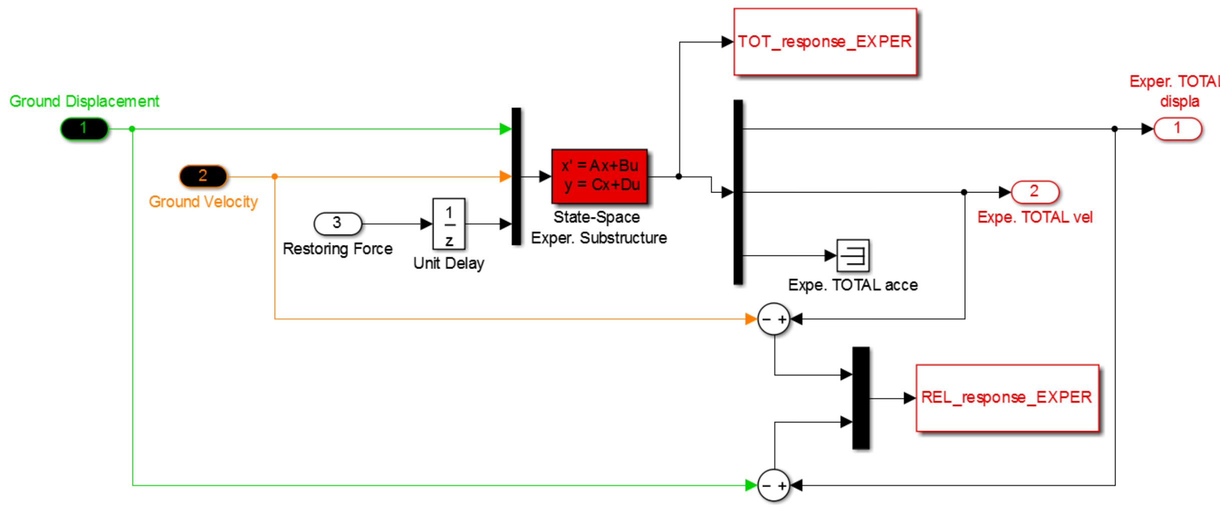

Figure 7. Ideal RTHS Experimental Substructure

Figure 7 above shows the setup of the experimental substructure in the ideal RTHS case, which does not include an actuator.

2.2.2 Numerical substructure

The numerically modeled steel frame in this demo has the same properties as the steel structure simulated in the original demo. Using Equation (2), the equation of motion for the numerical substructure, two first order state space equations are derived below.

$$\dot{z}_N = \begin{bmatrix} y_N \\ \dot{y}_N \end{bmatrix} = A_N \begin{bmatrix} x_N \\ y_N \end{bmatrix} + B_N \begin{bmatrix} x_E \\ y_E \end{bmatrix} \quad (12)$$

$$\ddot{z}_N = \begin{bmatrix} x_N \\ y_N \\ \dot{y}_N \end{bmatrix} = C_N \begin{bmatrix} x_N \\ y_N \end{bmatrix} + D_N \begin{bmatrix} x_E \\ y_E \end{bmatrix} \quad (13)$$

where,

$$A_N = \begin{bmatrix} 0 & 1 \\ -(k_N/m_N) & -(c_N/m_N) \end{bmatrix} \; ; \; B_N = \begin{bmatrix} 0 & 0 \\ (k_N/m_N) & (c_N/m_N) \end{bmatrix}$$

$$C_N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -(k_N/m_N) & -(c_N/m_N) \end{bmatrix} \; ; \; D_N = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ (k_N/m_N) & (c_N/m_N) \end{bmatrix}$$

```
% STATE SPACE MODEL(Numerical Substructure)
AA_N=[0 1; -KN/MN -CN/MN];
BB_N=[0 0; KN/MN CN/MN];
CC_N=[1 0; 0 1; -KN/MN -CN/MN];
DD_N=[0 0; 0 0; KN/MN CN/MN];
sys_N = ss(AA_N,BB_N,CC_N,DD_N);
```

These state space equations are defined through the script file, BI_RTHS_DM.m and then implemented in the BI_RTHS_simul.mdl Simulink file, as seen in Figure 8 below.

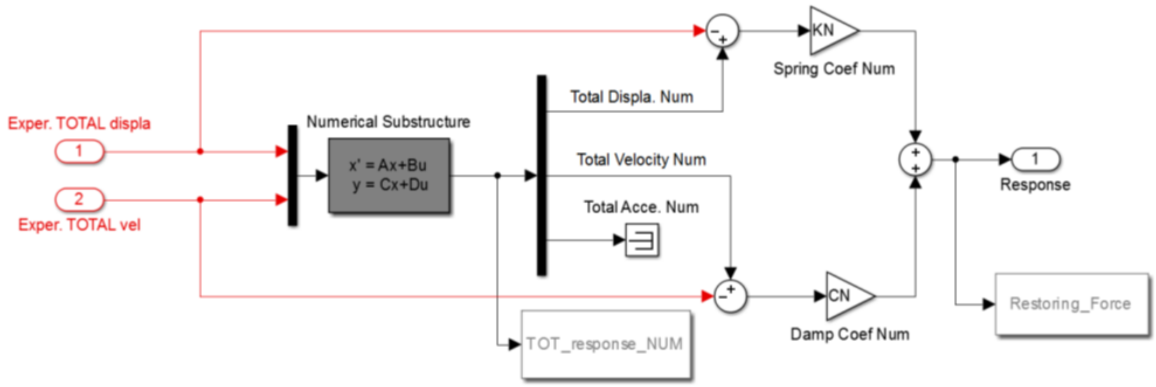

Figure 8. Ideal RTHS Numerical Substructure

2.3 Actuator Dynamics, Compensation Technique and Full RTHS

The full RTHS case includes an actuator in the cycle, as opposed to the ideal RTHS. It simulates the dynamics of the actuator, which must be considered in order to more accurately replicate real-world conditions. The dynamics of the actuator have not changed from that of the original demo (Ou, 2013) and the set up can be found in that report. Much like the original demo, the actuator has 8 different design sets to choose from.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%       Actuator Dynamics and Control       %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
C_sw = 4;                % actuator compensation switch choose from 1-8
load (['Level_' num2str(C_sw) '_comp.mat']);
load ACT_DYN;             % actuator dyanmcis
```

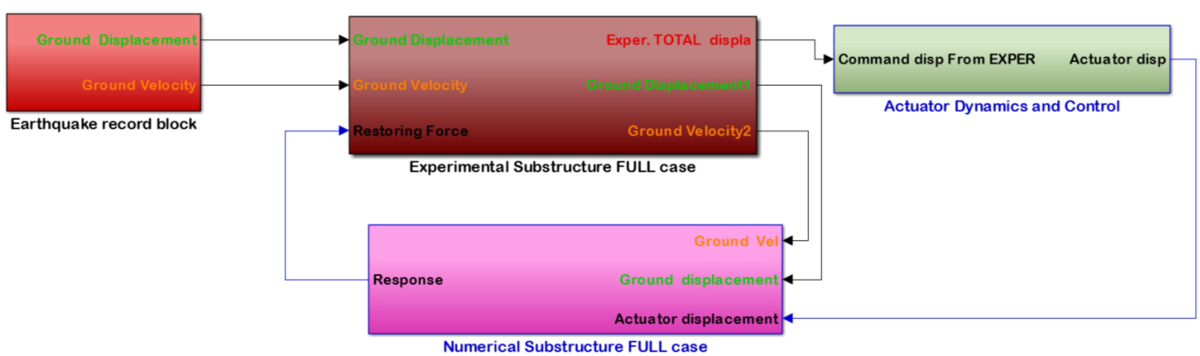

Figure 9. Full RTHS simulation concept

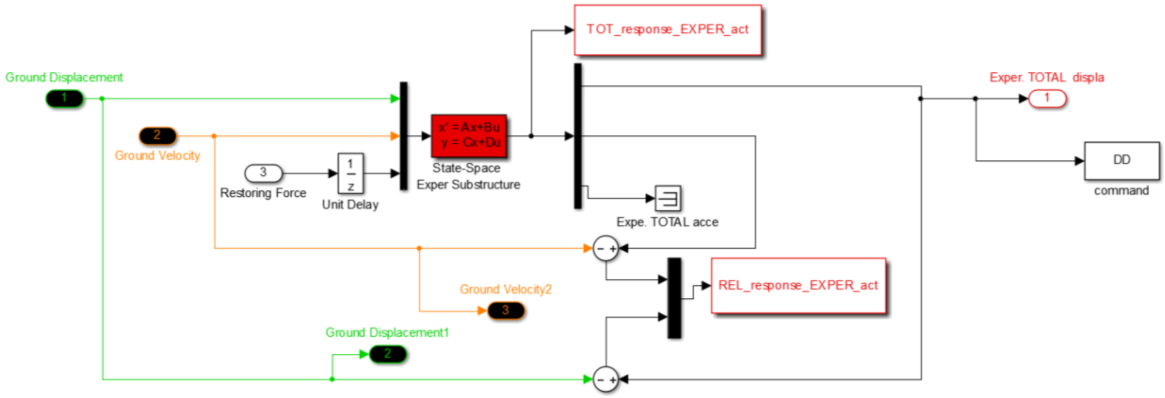

Figure 10. Experimental substructure full case

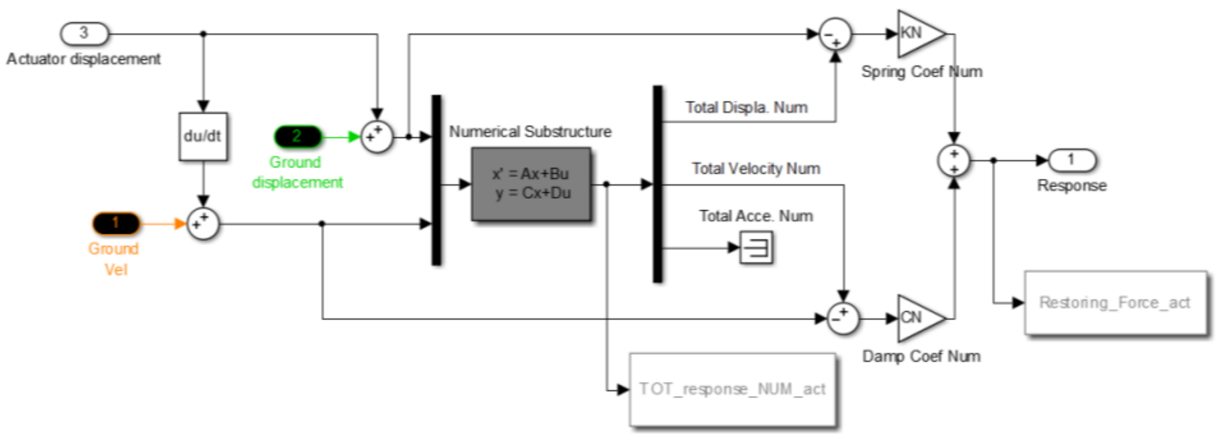

Figure 11. Numerical substructure full case

3 Simulation and sample output

In the simulation, the user may choose from different earthquake records: El Centro, Kobe and Morgan earthquake records. Earthquake intensity can be defined based on linear amplification of the record. The simulation time step is FIXED as 1/1024 sec. Simulation time differs based on the earthquake record chosen.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%      General Simulation Parameter         %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
dt_rths = 1/1024;

E_sw = 1;              % Earthquake switch 1: el centro 2: kobe 3: morgan
eq_intensity = 1;   % EQ Intensity

switch E_sw   % Earthquake Ending Time
case 1
    Tmax = 29.36;
case 2
    Tmax = 50.94;
case 3
     Tmax = 39.974;
end

OPTIONS = simset('solver','ode5','FixedStep',dt_rths);
sim('BI_RTHS_simul',[0 Tmax],OPTIONS,[]);
```

Upon execution of the RTHS, the code will plot responses. The first three figures plotted from BI_RTHS_DM.m are the experimental and numerical structural response (displacement, velocity and acceleration) comparisons between the reference case, ideal RTHS and full RTHS. The following figures show the tracking control performance of the actuator and restoring force in the ideal RTHS and full RTHS cases.

The following output plots are generated below based on settings in Table 2.

Parameter	**Value**
Base Isolation Option	1
C_sw	4
eq_intensity	1
E_sw	1

Table. 2 Sample Settings (default)

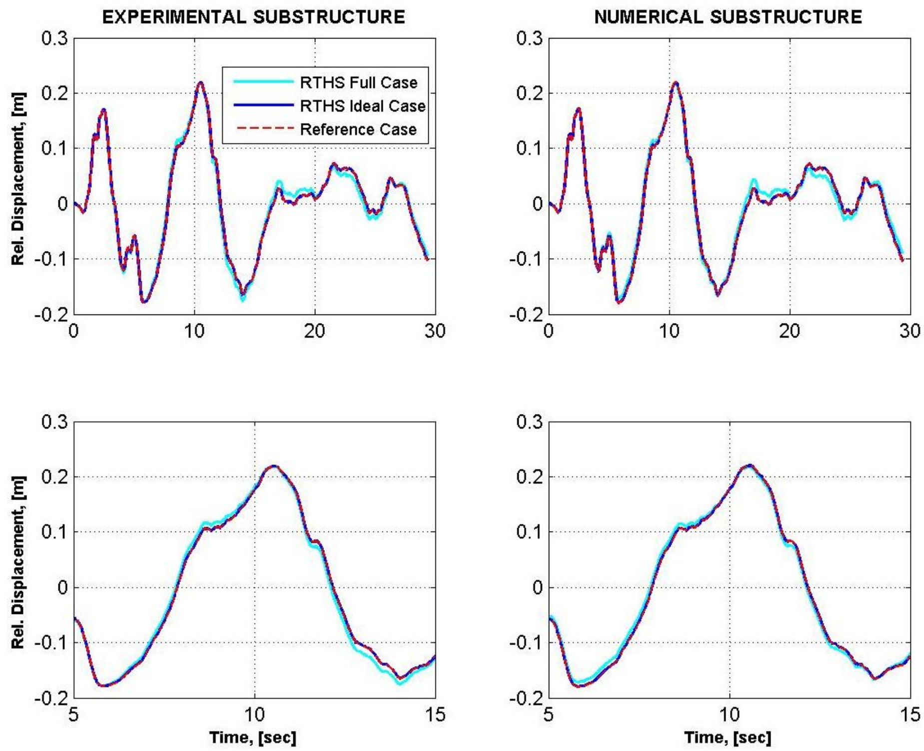

Figure 14. Displacement comparison between simulation, full RTHS, ideal RTHS

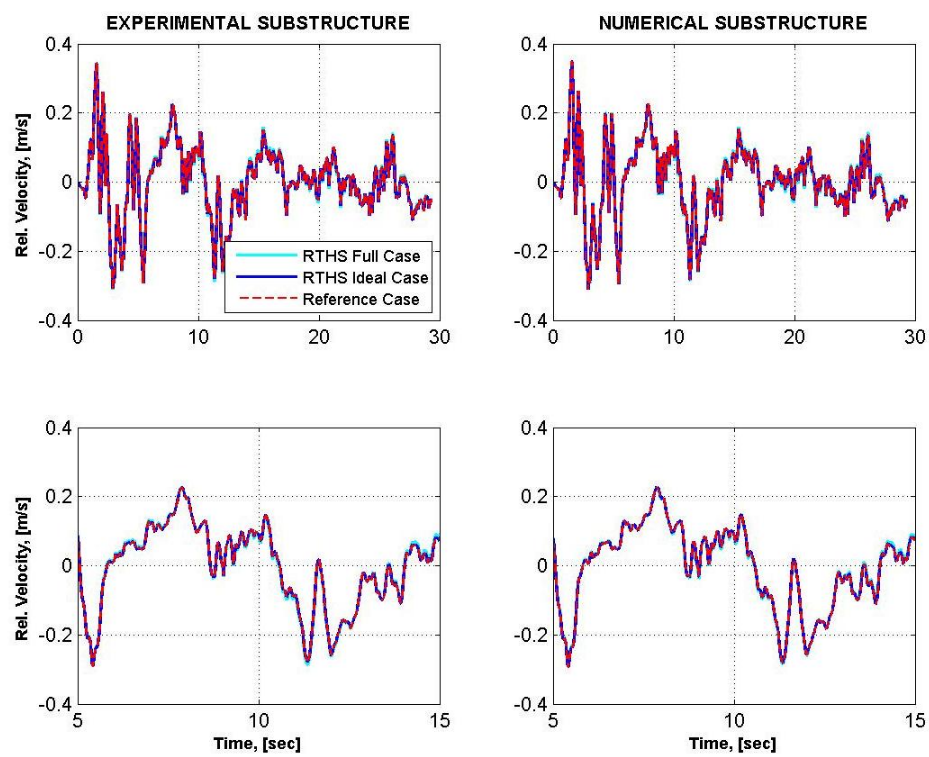

Figure 15. Velocity comparison between simulation, full RTHS, ideal RTHS

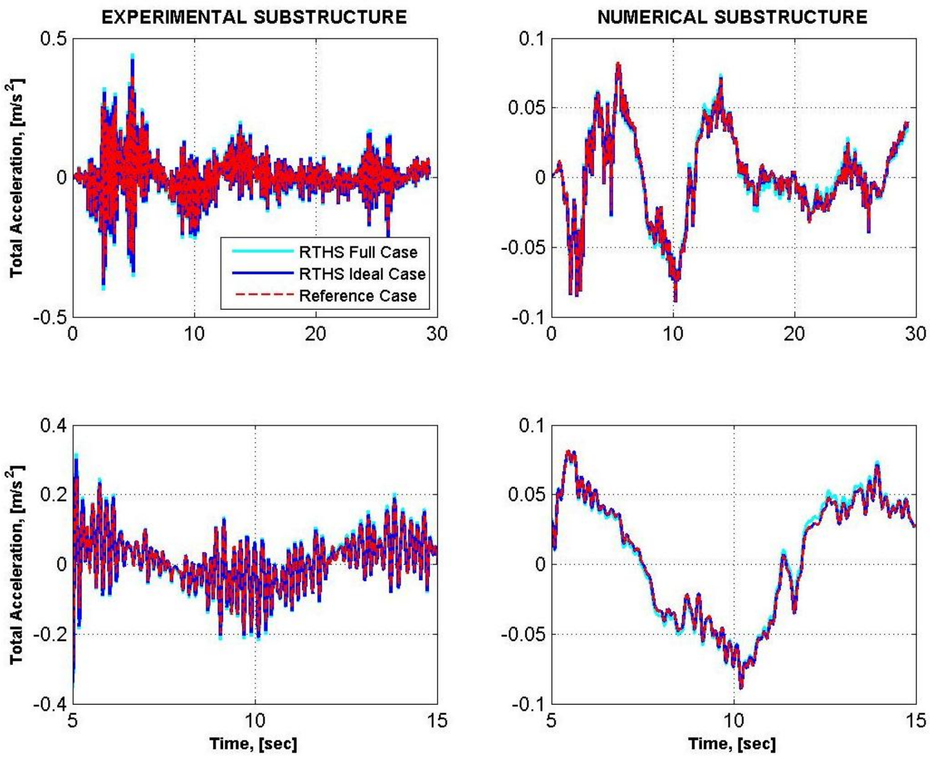

Figure 16. Acceleration comparison between simulation, full RTHS, ideal RTHS

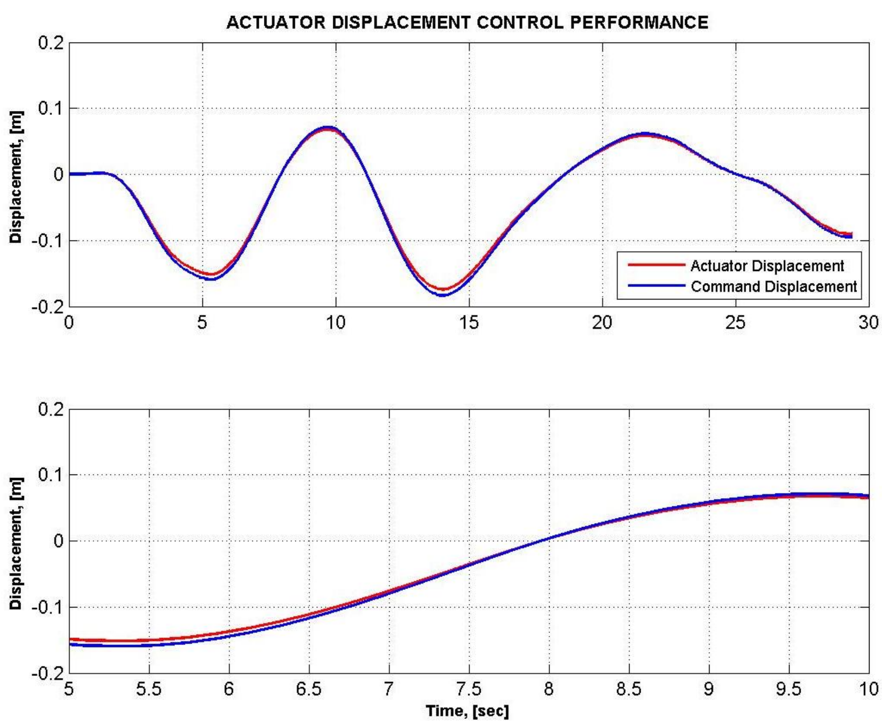

Figure 17. Actuator displacement control performance

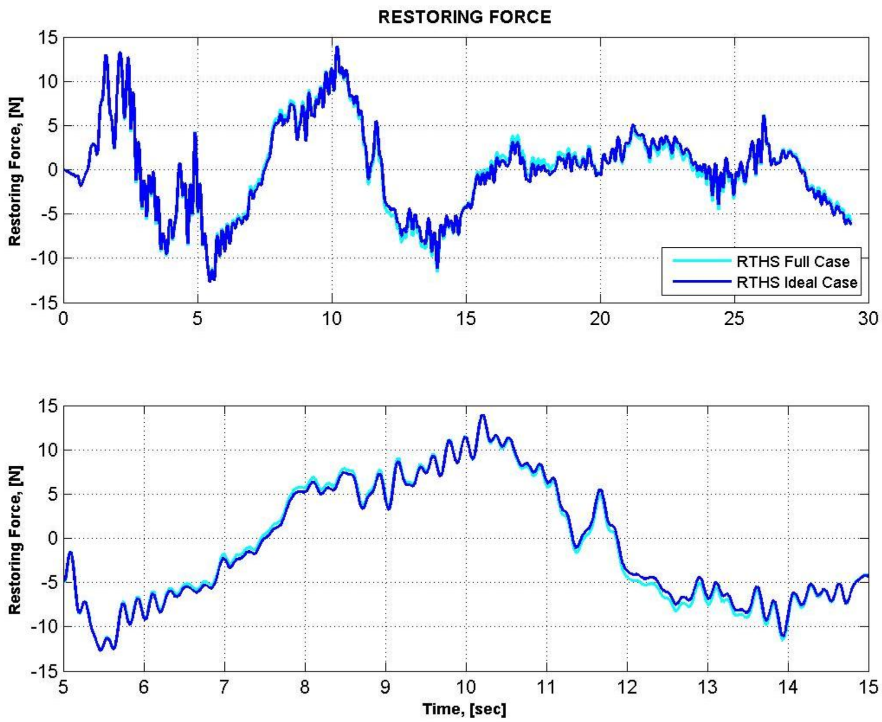

Figure 18. Restoring force comparison between full case and ideal case RTHS

4 Base Isolation comparison

In this demo, the ground motions were reported in terms of displacement and velocity that were obtained by numeric integration from ground acceleration. To validate the accuracy in the numerical integration, the relative response of the structure with base isolation subject to ground acceleration was compared with the response of the SDOF subject to ground displacement and ground velocity inputs. The SDOF responses (relative displacement, relative velocity, and total acceleration) due to El Centro ground motion are shown in Figure 19. The Matlab code and Simulink files are BI_compare_01.m and bi_sim_compare.mdl, respectively, and are included in this demo for your review.

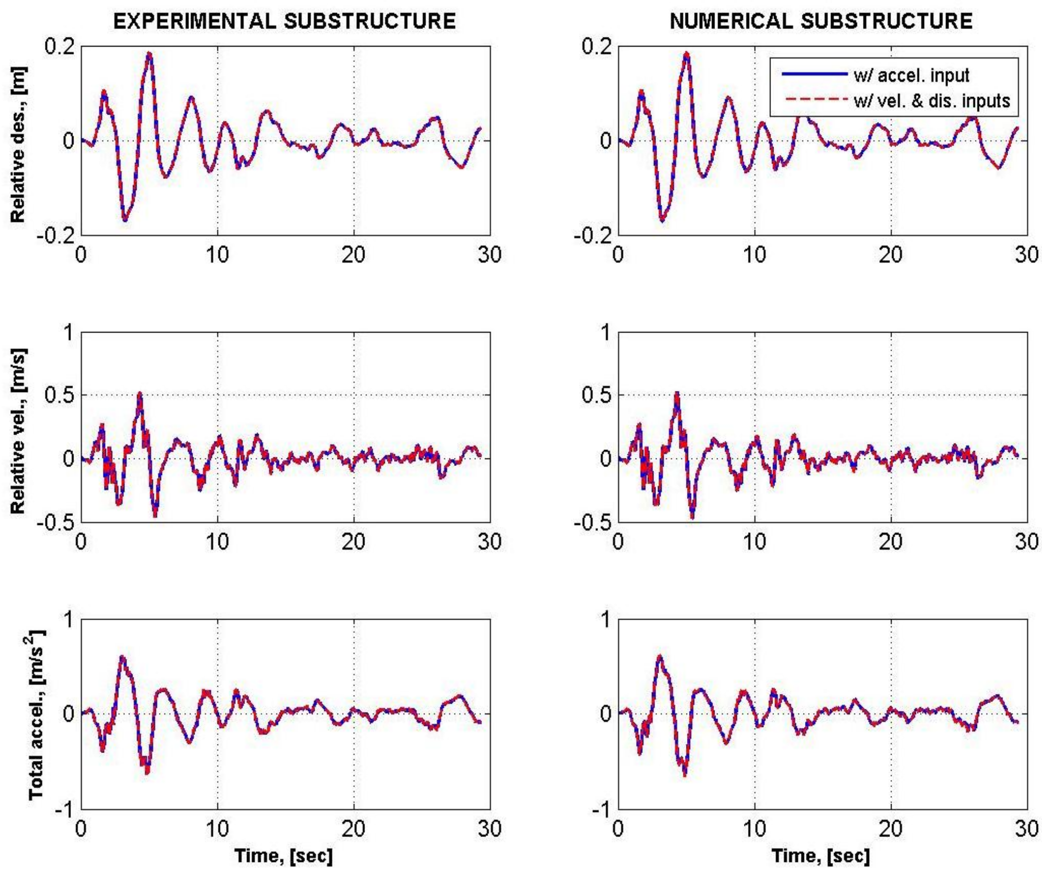

Figure 19. Relative displacement, Velocity and total (absolute) acceleration for experimental vs numerical substructures

5 Conclusion

This document, together with code 'BI_RTHS_simul.mdl' and 'BI_RTHS_DM.m', presents a simple RTHS simulation example, where both the numerical substructure and the experimental substructure are numerically modeled. The experimental substructure is a base isolation system and the numerical substructure is a one-story linear steel structure. The sample code divides the simulation into three parts: entire building response, ideal RTHS response and full RTHS response. As can be seen by the output graphs, all three simulations provide very comparable results. The default parameters shown in this document produce the output figures provided in order to make comparison possible.

QUESTIONS RELATED SHOULD BE DIRECTED TO DANIEL GOMEZ (EMAIL: dgomezp@purdue.edu).

Reference

Castaneda, N.E., Gao, X., Dyke, S.J. (2012). "Development and Validation of a Computational Tool for Real-time Hybrid Simulation of Steel Frame Structures", Report IISL — 002.

Ou, G., Dyke, S.J. (2013), "A Real Time Hybrid Simulation Demo Code," https://nees.org/resources/6984.

Maghareh, A., Dyke, S.J., Prakash, A, & Rhoads, JF. (2013) *Establishing Predictive Indicators for Stability and Performance of SDOF Real-Time Hybrid Simulations.* Purdue University. Report IISL – 004, November.

NEHRP, (2013). "When (Physical + Numerical = Testable System)", *Seismic Waves*, Dec, pp. 1-2.