A Real Time Hybrid Simulation (RTHS) DEMO for a Single Story Steel Structure[a]

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Contents

Abstract

This document includes description to an easy understandable RTHS simulation example including codes 'RTHS_DM.mdl' and 'Trigger_RTHS_DM.m'. In this example, the concept of RTHS and its representation in simulation is discussed. A single story steel structure with predetermined mass, stiffness, damping ratio is simulated herein, the structure is partitioned into the combination of one numerical substructure and one experimental substructure. The actuator used in RTHS is modeled with physical data and actuator displacement compensation is also simulated.

The purpose of preparing this demo example is to help beginners to understand RTHS concept. To be able to run the code, user can jump to section 3 without difficulty. This activity was supported in part by the US National Science Foundation (NSF) under Award Number CMMI-0927178.

Keyword: Hybrid Simulation, Real Time Hybrid Simulation, Demo code

[a] **NOTE: This code is the simulation of RTHS and does not run in real time machine.**

1 Concept of RTHS

Hybrid simulation splits the entire structure into two parts, one is the well understood portion that can be modeled in simulation and the other is the more complex portion which needs to be evaluated though experiment. Real time hybrid simulation is one state-of-art expansion of hybrid simulation which facilitates investigating a structure's dynamic response in real time including rate dependent effect.

One typical RTHS cycle includes: **a)** send seismic loading numerically through equation of motion in simulation, then **b)** applies the calculated structural response from step (a) though actuator in real time (mostly beyond 1kHz scale) and **c)** send back the measured restoring force from experimental substructure to simulation and preparing the next cycle. Figure 1 shows a seismic study of a four story frame under RTHS partition.

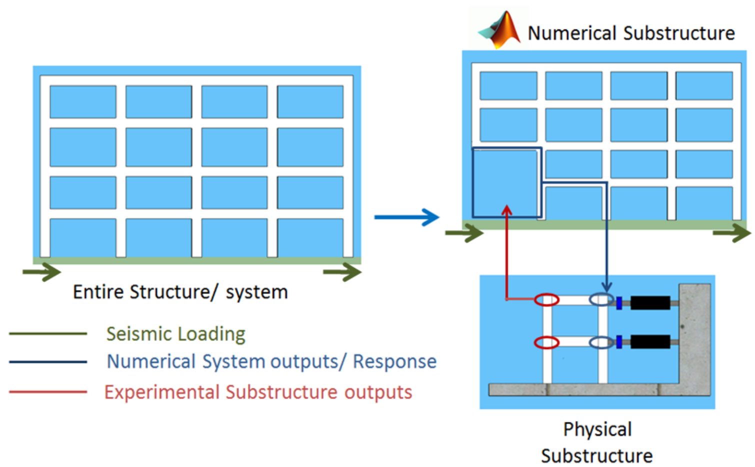

Figure 1. Concept of RTHS using a four story four bay frame[b]

2 Structure of demo code

The demo code is consists of two major files, RTHS_DM.mdl and Trigger_RTHS_DM.m. RTHS_DM.mdl is a Matlab Simulink® code that allows simulation in block diagram environment. It is compatible with Simulink 6.6/R2007a version or later. This code has the interface shown in Figure 2. Another file Trigger_RTHS_DM.m has all the predefined parameter and executes the Simulink code.

RTHS_DM.mdl is divided into three sections. First is total structure response under earthquake (yellow), no partitioning is applied here to allow for comparisons. The second part is to perform an ideal RTHS simulation, without considering any actuator dynamics or actuator control, here, the entire structure is separated into numerical substructure (grey) and experimental substructure (dark red). The last section is a relative full simulation for RTHS, where additional actuator dynamics has been modeled (green) and well compensated.

[b] **NOTE: This figure is only conceptual, for demo code and example, only a SDOF structure is considered.**

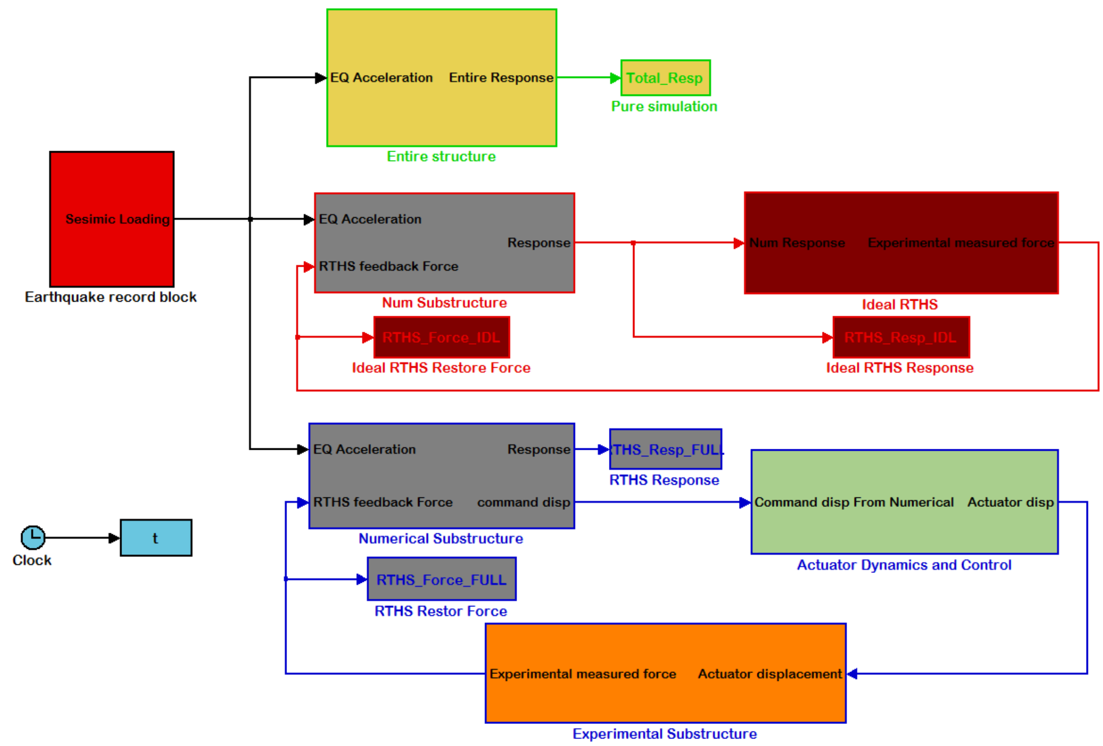

Figure 2. RTHS_DM.mdl Simulink Diagram

In the following sections, each of the example components is explained in more detail.

2.1 Structural properties and state space form

In this demo code, only the linear frame problem is considered. Thus, the entire structure mass and stiffness are both constant under seismic loading.

$$M_T = 58.6\frac{N}{m^2/s}, K_T = 6.05 \times 10^4\frac{N}{m}, \zeta_T = 0.02$$

where M_T, K_T, ζ_T are mass, stiffness, damping ratio of the entire structure, respectively.

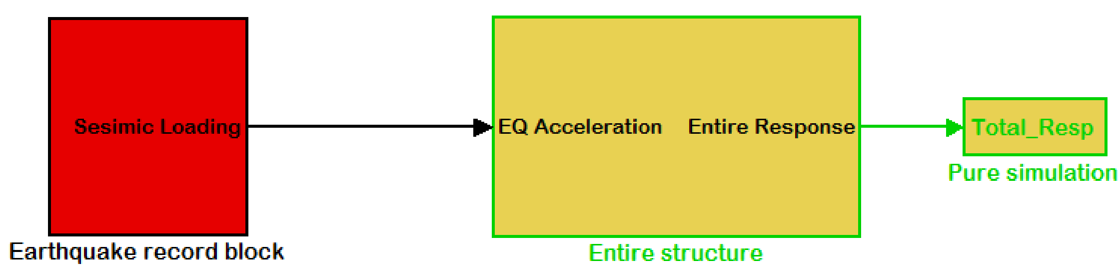

Figure 3. Entire structure response simulation (no partitioning)

Simulink takes care of all numerical integration and the easiest way to simulate a linear structure response under seismic loading is to use state space form of motion equation.

$$M_T\ddot{x} + C_T\dot{x} + K_T x = -M_T\ddot{x}_g \quad (1)$$

Thus, the 2nd order equation of motion is written as two first order equations as:

$$\dot{X} = A_T X + B_T u \tag{2}$$

$$y = C_T X + D_T u \tag{3}$$

Where,

$$A_T = \begin{bmatrix} 0 & 1 \\ -\frac{K_T}{M_T} & -\frac{C_T}{M_T} \end{bmatrix}; B_T = \begin{bmatrix} 0 \\ -1 \end{bmatrix}; C_T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -\frac{K_T}{M_T} & -\frac{C_T}{M_T} \end{bmatrix}; D_T = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

Eqn. (2) and (3) is the state space representation of Eqn (1), where $u = \ddot{x_g}$, is the earthquake excitation, $X = [x; \dot{x}]$ is the state vector, and $y = [x; \dot{x}; \ddot{x}]$which is structural response vector including displacement, velocity, acceleration, respectively.

The corresponding parameters are provided in Trigger_RTHS_DM.m file.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%       Entire Building Information         %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
M    = 155.6;                       % mass per floor [N/(m/sec2)]
K       = 1.004922e+05 ;                    % N/m
Zeta   = 0.02;
C=2*Zeta*sqrt(K*M);
wn_T=sqrt(K/M)/(2*pi());
wd_T=wn_T*sqrt(1-Zeta^2);

disp(['Entire Structure Natural Frequency is ' num2str(wd_T) ' Hz'])

AA_T=[0 1; -K/M -C/M];
BB_T=[0; -1];
CC_T=[1 0; 0 1; -K/M -C/M];
DD_T=[0; 0; -1];
```

2.2 RTHS partitioning, numerical substructure and experimental substructure

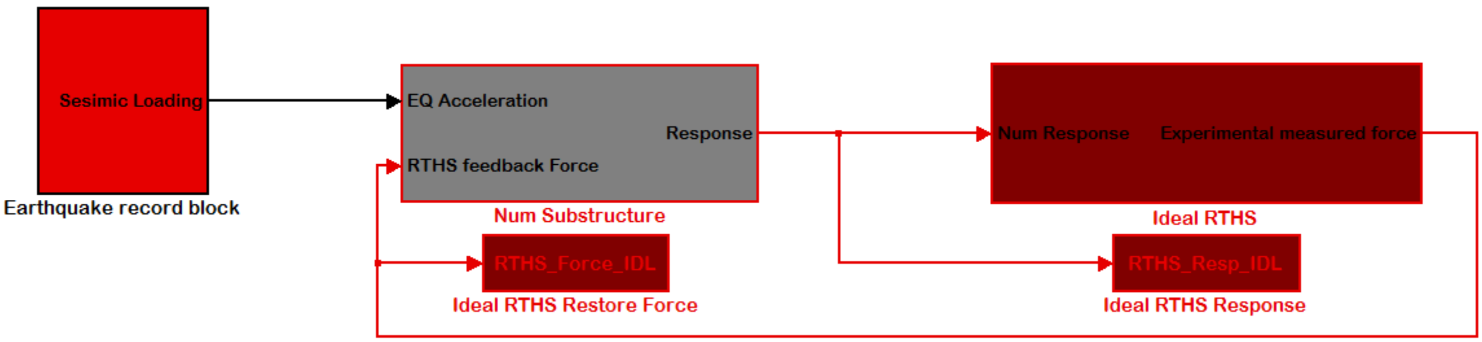

Figure 4. Ideal case RTHS

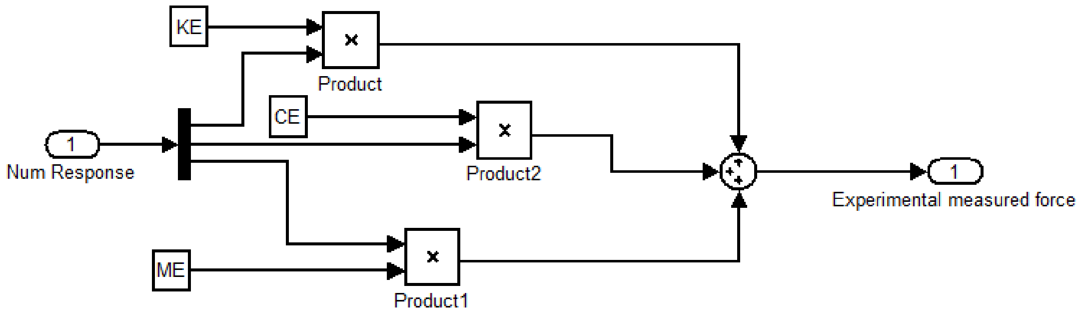

Figure 5. Ideal RTHS Experimental substructure

In the second section of RTHS_DM.mdl, ideal RTHS case is simulated where actuator dynamics is not considered thus the numerical response is applied to experimental substructure ideally. For this simulation, we rewrite the equation of motion as:

$$(M_E + M_N)\ddot{x} + (C_E + C_N)\dot{x} + (K_E + K_N)x = -M_T\ddot{x}_g \tag{4}$$

$$M_E\ddot{x} + C_E\dot{x} + K_E x = R \tag{5}$$

$$M_N\ddot{x} + C_N\dot{x} + K_N x = -M_T\ddot{x}_g - F \tag{6}$$

$$M_T = M_E + M_N, C_T = C_E + C_N, K_T = K_E + K_N$$

As before, Eqn. (4-6) can be written in state space form as:

$$\dot{X}_{RTHS} = A_N X_{RTHS} + B_N u_{RTHS} \tag{7}$$

$$y_{RTHS} = C_N X_{RTHS} + D_N u_{RTHS} \tag{8}$$

$$A_N = \begin{bmatrix} 0 & 1 \\ -\frac{K_N}{M_N} & -\frac{C_N}{M_N} \end{bmatrix}; B_N = \begin{bmatrix} 0 & 0 \\ -\frac{M_T}{M_N} & -\frac{1}{M_N} \end{bmatrix}; C_N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -\frac{K_N}{M_N} & -\frac{C_N}{M_N} \end{bmatrix}; D_N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -\frac{M_T}{M_N} & -\frac{1}{M_N} \end{bmatrix}$$

where, subscript N represents the numerical portion, subscript E represents the experimental portion. $u_{RTHS} = [\ddot{x}_g \; R]^T$ contains two inputs including the earthquake excitation and restoring force from the experimental substructure.

From Figures 4 and 5, the earthquake loading is applied to the numerical substructure (grey) first, and the numerical response is ideally imposed to the experimental substructure (dark red), the induced restoring force is fed back to the numerical substructure. Trigger_RTHS_DM.m allows users to change mass and stiffness distribution between numerical substructure and experimental substructure. **ATTENTION: Based on the stability requirements, it is not suggested to put more than 35% mass in to the experimental substructure in this demo code.**

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%             Patitioning of RTHS             %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
R_ME=0.15; %ratio of floor mass in experimental substructure
% not suggested over 0.35
R_KE=0.5; %ratio of floor stiffness in experimental substructure
Zeta_E=0.0045; %damping ratio of first story

% CHECK STABILITY OF Partition
if (R_ME > 0.35)
    disp(' ')
    disp('Unacceptable Mass Partition !')
end

ME=R_ME*M;
KE=R_KE*K;
CE=2*Zeta_E*sqrt(KE*ME);
wn_E=sqrt(KE/ME)/(2*pi());
wd_E=wn_E*sqrt(1-Zeta_E^2);

MN=M-ME;
KN=K-KE;
CN=C-CE;

AA_N=[0 1; -KN/MN -CN/MN];
BB_N=[0 0; -M/MN -1/MN];
CC_N=[1 0; 0 1; -KN/MN -CN/MN];
DD_N=[0 0; 0 0; -M/MN -1/MN];
```

2.3 Actuator Dynamics, Compensation Technique and Full RTHS

In the RTHS realization, the numerical response is applied to the experimental substructure using an actuator. Intrinsically, actuators have dynamics and thus there are phase lags and amplitude attenuation (Dyke et al, 2005). These effects should not be ignored. They require the use of an appropriate control techniques and careful analysis of the test setup to ensure a safe and effective test is performed.

The third section of RTHS_DM.mdl includes actuator dynamics and control algorithm. The actuator model is retrieved from a physical experimental setup located in School of Civil Engineering, Harbin Institute of Technology, China.

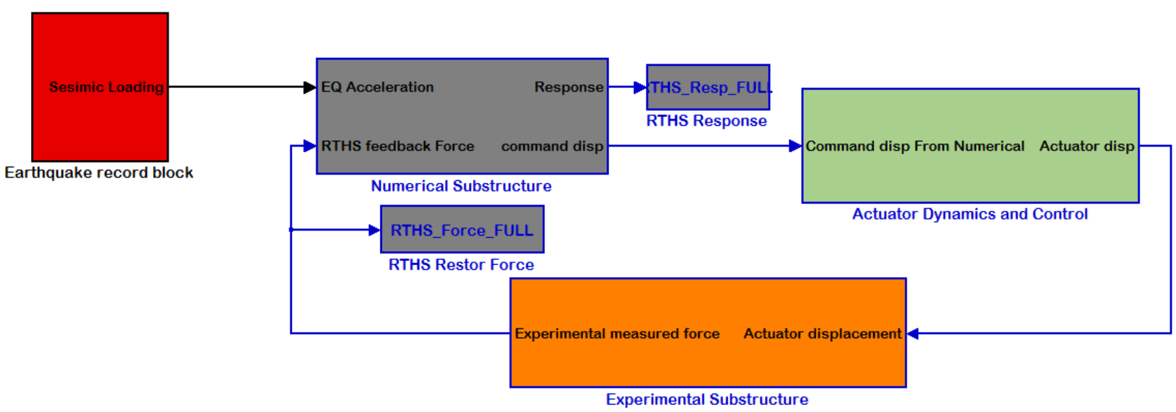

Figure 6. Full RTHS simulation concept

For the simulation, actuator is modeled as a 4th order transfer function based on a frequency domain response curve fitting approach. A BLWN displacement input is sent to actuator and the response is recorded, to determine the experimental transfer function. Details on the actuator identification method can be found in (Carrion & Spencer, December 2007; Phillips & Spencer, June 2011). Figure 7 shows a comparison of the identified model and the experimental data.

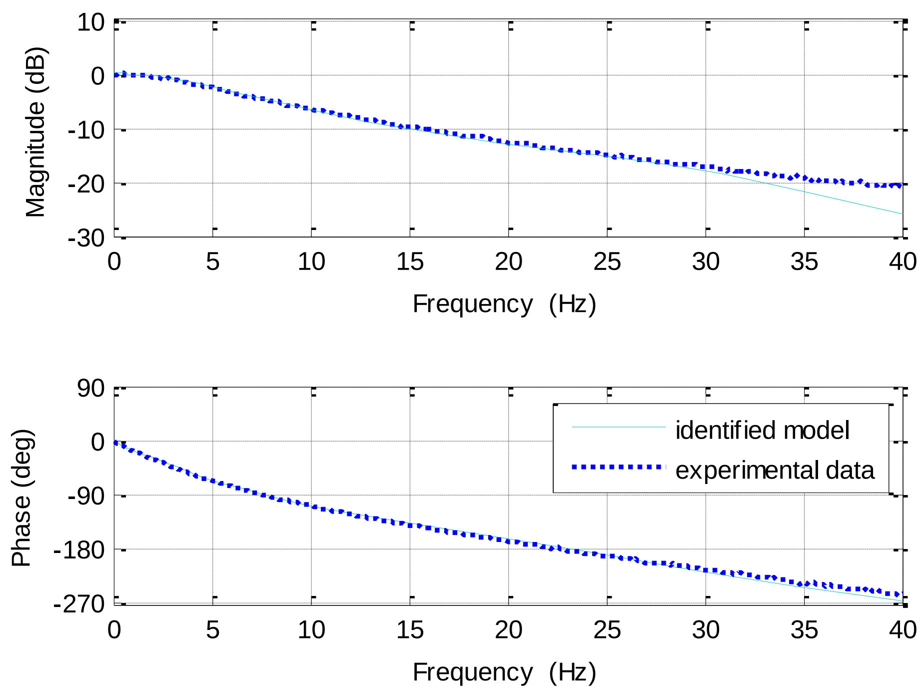

Figure 7. Actuator model identification result

From Figure 7, it is clear that above 2 Hz, the actuator transfer function has magnitude attenuation, and 180° phase lag observed at 20 Hz. Thus, actuator control is needed to compensate for the dynamics and achieve the desired displacement from the actuator. In this demo code, one H_∞ based actuator control algorithm is used as shown in Figure 8. Details of the design can be found in (Ou et al., June 6-9, 2012; Ou, et al. May 28-30, 2013)

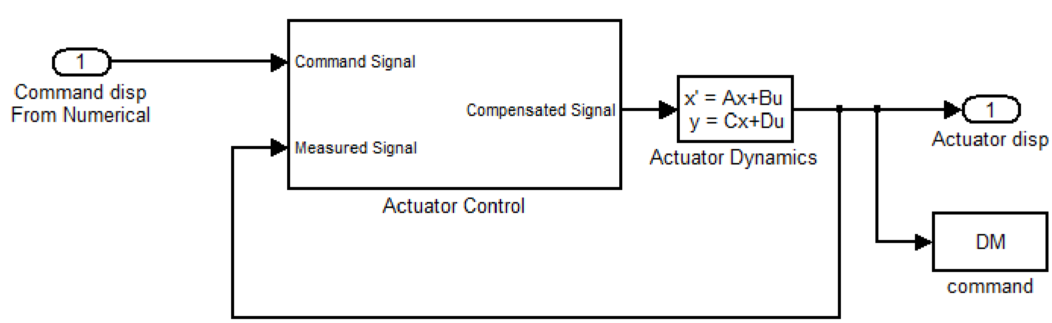

Figure 8. Full RTHS, actuator dynamics and compensation

Instead of directly applying the numerical response (including calculated displacement, velocity, acceleration) to the experimental substructure as in an ideal RTHS simulation, we use derivatives of actuator displacement to calculate the experimental substructure's velocity and acceleration.

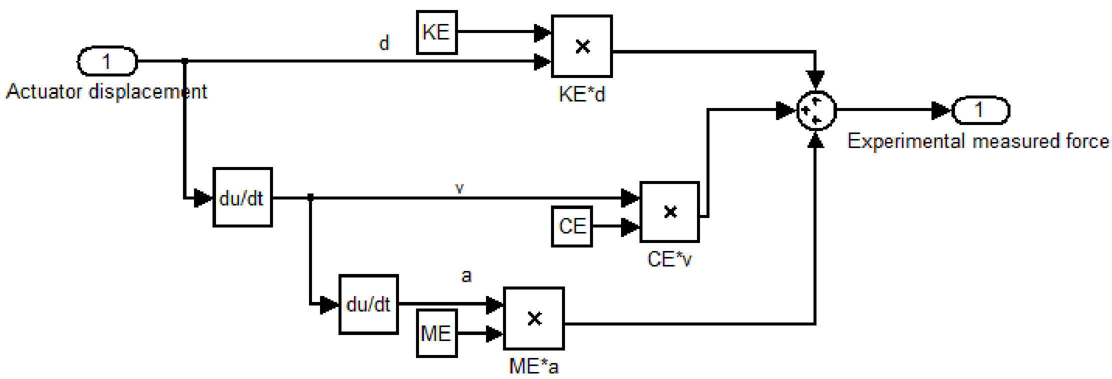

Figure 9. Full RTHS, force generation (inside Experimental Substructure)

Trigger_RTHS_DM.m file allows users to choose from 8 different actuator control design sets and load actuator dynamics from predefined files.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%      Actuator Dynamics and Control        %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
C_sw = 4;               % actuator compensation switch, choose from 1-8
load (['Level_' num2str(C_sw) '_comp.mat']);
load ACT_DYN;            % actuator dyanmcis
```

3 Simulation and sample output

In the simulation, the user may choose from: different earthquake records, El Centro, Kobe and Morgan earthquake records. Earthquake intensity can be defined based on linear amplification of the record. The simulation time step is FIXED as 1/1024 sec, simulation time length is 80 sec.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%      General Simulation Parameter          %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
dt_rths = 1/1024;
eq_intensity = 1;   % EQ Intensity
E_sw = 1;             % Earthquake switch 1: el centro 2: kobe 3: morgan
tend=80;              % Earthquake Ending Time
sim ('RTHS_DM')
```

Upon execution of the RTHS, the code will plot responses. The first three figures plotted from Trigger_RTHS_DM.m are the structural response (displacement, velocity and acceleration) comparison between entire structure, ideal RTHS and full RTHS. Next figures show the tracking control performance of the actuator and restoring force in ideal RTHS and full RTHS.

The following output plots are generated under settings in Table-1.

Table-1 Sample Settings (default)

Parameter	Value
R_ME	0.15
R_KE	0.15
C_sw	4
eq_intensity	1
E_sw	1

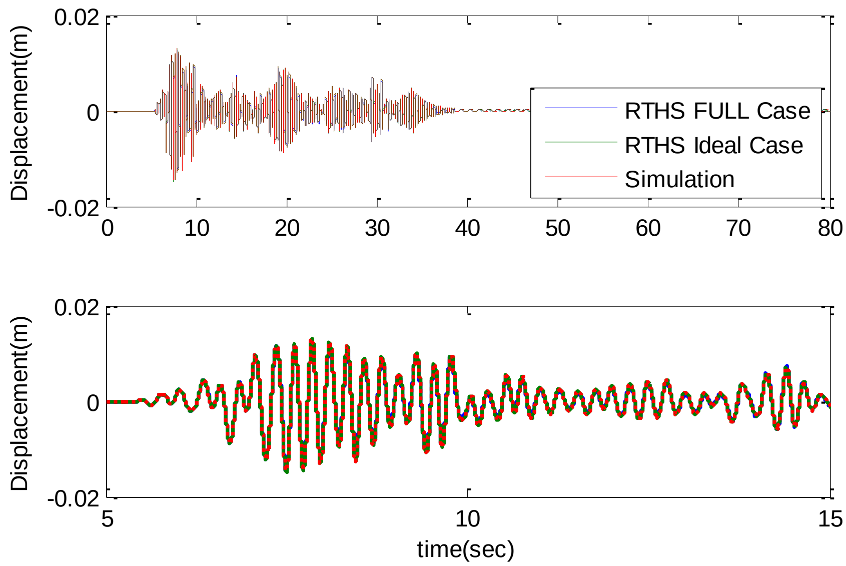

Figure 10. Displacement compensation between simulation, full RTHS, ideal RTHS

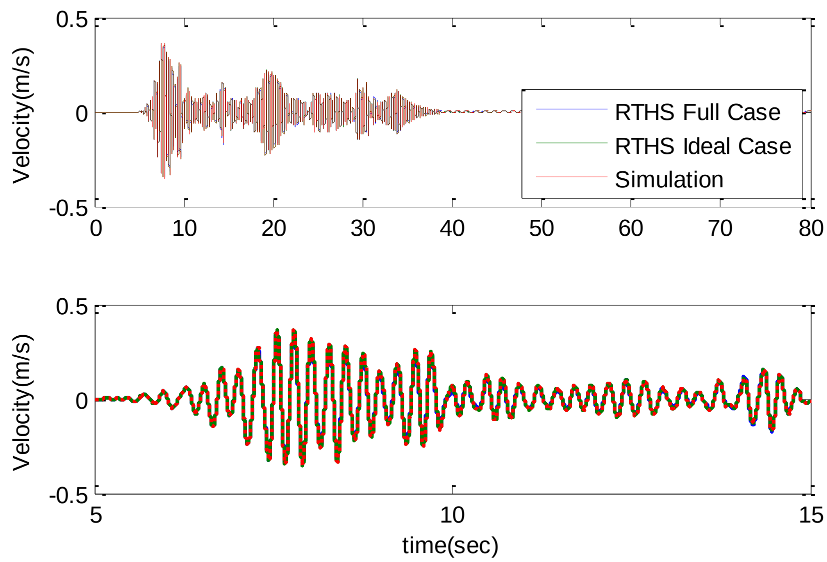

Figure 11. Velocity compensation between simulation, full RTHS, ideal RTHS

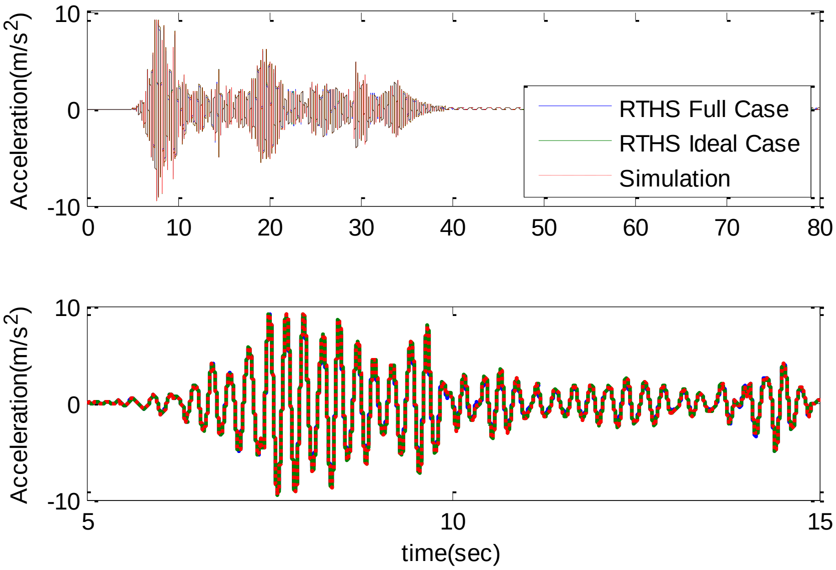

Figure 12. Acceleration compensation between simulation, full RTHS, ideal RTHS

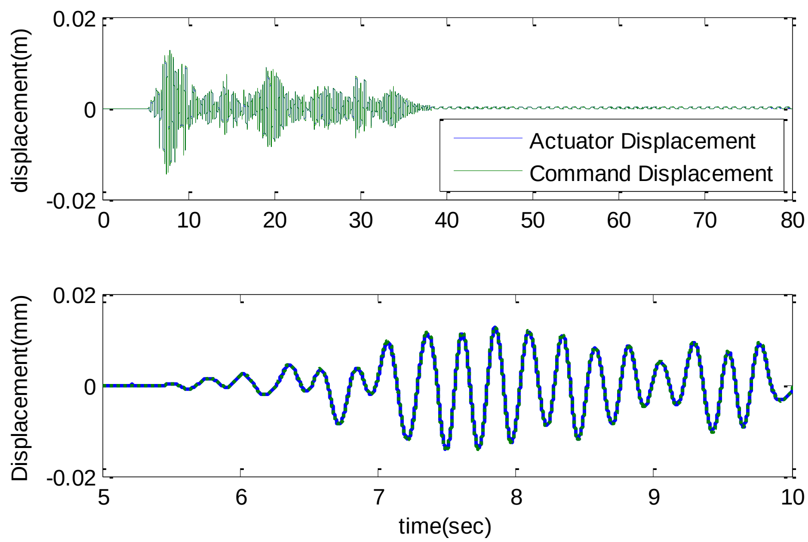

Figure 13. Actuator Displacement Control Performance

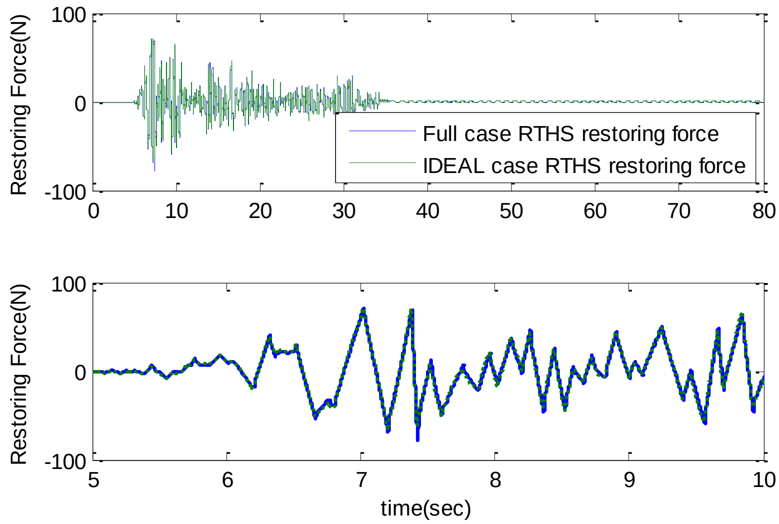

Figure 14. Restoring force comparison between full case and ideal case RTHS

4 Conclusion

This document, together with code 'RTHS_DM.mdl' and 'Trigger_RTHS_DM.m', presents a simple RTHS simulation example using a linear single story frame, where both the numerical substructure and the experimental substructure are numerically modeled. The sample code divides the simulation into three parts: entire building response, ideal RTHS response ad full RTHS response. The code allows user to choose different partitioning for mass and stiffness distribution between numerical portion and experimental portion. Output with given parameters are presented for user to compare if necessary.

This activity was supported in part by the US National Science Foundation (NSF) under Award Number CMMI-0927178.

QUESTIONS RELATED SHOULD BE DIRECTED TO GE (GABY) OU (EMAIL: gou@purdue.edu).

Reference

Carrion, J., & Spencer, B. (2007) *Model-based Strategies for Real-time Hybrid Testing.* University of Illinois at Urbana-Champaign: NSEL Report Series., December.

Dyke, S.J., Spence, B.F., Quast, P., & Sain, M.K., Role of Control-Structure Interaction in Protective System Design. *Journal of Engineering Mechanics*, ASCE, Vol. 121 No. 2, pp. 322–38, 1995

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